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MTH 530, Abstract Algebra I (graduate) Fall 2012 ,HW number SIX (Due: Sat. at 1pm November 24)

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QUESTION 1. (i) Let $a \in A$ for some group A and suppose that H is a subgroup of A. Prove that $a^{-1}Ha = \{a^{-1}ha | h \in H\}$ is a subgroup of A. In particular, if H is finite, then show that $|a^{-1}Ha| = |H|$.

(ii) Let M be a finite subgroup of a group K and $|M| = i$. Suppose that M is the only subgroup of K that has order i. Prove that M is a normal subgroup of K.

(iii) Let M, i, and K as in (ii) (also M is the only subgroup of K of order i). Suppose that $|K| = ij$ where $gcd(i, j) = 1$. Let W be a subgroup of K of order c such that $c \mid i$. Prove that W is a subgroup of M.

(iv) Given D is a group and $|D| = 7^3 \times 11^2 \times 5$. Assume that D is solvable. Prove that D has a unique normal subgroup of order $7^3 \times 11^2$.

(v) Assume that F is a simple group with odd order m. Given $13 \mid m$. Can you tell what is F?

(vi) Let $M = Z_3 \oplus Z_3 \oplus Z_9$. Construct a composition series of M.

(vii) Let F be a group and $|F| = 13 \times 77$. If F has a normal subgroup of order 11, then prove that F has a normal subgroup of order 143.

(viii) If H, K are subgroups of a group D, then we know that HK needs not be a subgroup of D. However, let H, K be subgroups of a group D. Prove that HK is a subgroup of D if and only if $HK = KH$.

Faculty information

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